# Cosmological Implications of the Planck Aether Model for a Unified Field Theory

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In the previously proposed Planck aether model of a unified theory, the vacuum ground state consists of a dense assembly of positive and negative Planck masses obeying an exactly nonrelativistic law of motion. The Planck aether is superfluid, and it can without expenditure of energy form a tangle of quantized vortex filamants permitting the transmission of two types of waves: One associated with a symmetric displacement of the vortex lattice leading to gravitational waves, and one associated with an antisymmetric displacement leading to electromagnetic waves. Dirac spinors are explained in this model as excitons made up from positive and negative energy resonances of the vortex lattice.

Because the number of positive and negative Planck masses is assumed to be equal, the cosmological constant is equal to zero. With the Dirac spinors formed as bound states from the Planck aether, the sum of the positive kinetic energy and negative gravitational energy must remain exactly equal to zero, resulting in $\Omega = 1$ as the exact value for the cosmological mass parameter.

To make up for the unobserved missing mass, estimated to be about 10 times larger than the baryonic mass, it is conjectured that this mass consists of rotons, which in the superfluid Planck aether would come from the cut-off of the energy spectrum near the Planck energy. Because of their unusual dispersion relation, rotons possess a large momentum, even if their velocity vanishes. They are, for this reason, a promising candidate for the nonbaryonic dark matter, combining properties of hot and cold dark matter. The critical value $\Omega = 1$ requires that the number density of the rotons is equal to $n_r \simeq 2 \times 10^{-25}$ cm$^{-3}$. This value corresponds to an average distance of separation between the rotons of the order $n_r^{-1/3} \simeq 6000$ km. The small number of rotons per unit volume combined with their weak gravitational coupling constant would make it difficult to detect them directly, but the gravitational field these roton masses generate in the vicinity of galaxies could explain the observed flat rotation curves of disc galaxies.

Finally, because the gravitational waves have a cut-off at the vortex lattice scale at $10^{15}$–$10^{16}$ GeV, there can be no singularity in the course of a gravitational collapse, as it happens for solutions of Einstein's gravitational field equations. A generalization of Einstein's gravitational field Lagrangian, taking into account the existence of a smallest wave length, rather predicts a conversion of all mass into electromagnetic (or gravitational) radiation in approaching the singularity. The predicted conversion of mass into radiation in the course of gravitational collapse may provide an explanation for the large energy release of quasars.

## 1. Introduction

There is growing evidence that most of the matter in the universe is nonbaryonic and cold [1]. The measured deuterium abundance, compared with the value obtained by detailed calculations [2] in the framework of the standard "big-bang" cosmology, gives as an upper limit for the baryonic mass a value about 10 times smaller than what is required for a flat universe with the critical mass density parameter $\Omega = 1$. The mass to light ratio (M/L) set by the abundances of deuterium and other light elements, gives a value for the dark unseen matter which is substantially smaller than the value obtained from the dynamic behavior of clusters of galaxies, independently suggesting that most of the missing mass must be nonbaryonic [1]. Additional dynamic evidence for the existence of nonbaryonic dark matter is provided by the flat rotation curves of disc galaxies [5]. Also, the observed clumpiness in the cosmic mass distribution favoring $\Omega \simeq 1$, is supported by computer simulations for the formation of galaxies and their distribution in space [6]. The inhomogeneities in the distribution of the galaxies can in these models not be obtained for a value $\Omega \simeq 0.1$, as-

[1] As an alternative explanation it has been suggested [3, 4] that Newton's law of gravitation is invalid for large distances. We believe that such a drastic hypotheses should be considered only as a last resort.

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suming that all matter is baryonic. Computer simulations made under the assumption that the missing nonbaryonic mass is hot, consisting for example of nonzero rest mass neutrinos, leads to the observed inhomogeneities, but only after a time much longer than the age of the universe.

Independent support for $\Omega = 1$ comes from general relativity. The counting of galaxies as a function of their distance, and assuming that the cosmological constant is zero, provides a simple method to determine the curvature of space. A detailed study done by Loh and Spillar [7] along these lines leads to a value indistinguishable from $\Omega = 1$. The assumption that $\Omega$ is exactly equal to one and the cosmological constant is exactly equal to zero is also from a purely esthetic point of view the most plausible hypothesis.

There is no lack of proposals for candidates making up the missing mass, but none of them is satisfactory. Some of the more serious candidates which have been proposed are: 1) axions; 2) primordial mini-black holes and 3) cosmic strings. The existence of axions, hypothetical particles which have been invented to account for CP violation, would have profound (but not observed) repercussions in the evolution of stars, except for the improbable coincidence that their mass happens to be within a small range. Combined with the lack of any observations of axions in the laboratory, doubts must be cast on their existence. Primordial mini-black holes are unlikely to exist in the number required to make $\Omega = 1$, because such mini-black holes would by Hawking radiation evaporate under the emission of gamma ray bursts at a rate which is not observed. Cosmic strings are even less likely to exist because they would lead to observable departures from the Planck distribution of the cosmic microwave spectrum [2].

A popular model suggesting that $\Omega = 1$, or at least that $\Omega$ should be very close to this value, is the inflationary model. Its advantage is that it gives preference from the many possible cosmological solutions of Einstein's field equations to those for which $\Omega \simeq 1$, but it provides no information about the nature of the missing baryonic mass. The distinguished value $\Omega \simeq 1$ is explained to result from not yet proven grand unified theories of elementary particle physics (involving the hypothetical Higgs field), rather than from a fundamental physical principle. In fact, it has been shown by Hübner and Ehlers [8] that even though inflationary models for which $\Omega = 1$ are very plausible, inflationary models in which $\Omega$ differs from $\Omega \simeq 1$ are possible as well. A more serious defect of inflationary models in general was pointed out by Penrose [9], who showed that they can not possibly work unless the Weyl curvature vanishes near the singularity, which is a very severe constraint.

In the face of these many difficulties we show that the recently proposed Planck aether model for a unified field theory can give a plausible explanation for these various puzzles. This model assumes that the fundamental kinematic symmetry in nature is the Galilei group, broken below the Planck scale into the Lorentz group, with the Lorentz group demoted into a dynamic symmetry [3]. The observed distribution and relative velocities of the galaxies suggest the existence of a distinguished reference system, making it plausible that the matter in the universe has been generated by a fundamental field at rest relative to this distinguished reference system.

## 2. The Planck Aether Model and Some of its Consequences

In the Planck aether model all elementary particles and interactions are derived from a fundamental nonlinear operator wave equation, very much as in Heisenberg's nonlinear spinor theory. Whereas in Heisenberg's model this equation is exactly relativistic, in the Planck aether model it is exactly nonrelativistic. And, whereas Heisenberg's relativistic model leads to unavoidable divergences, the nonrelativistic Planck aether model remains finite in all orders. The Planck aether model can in its groundstate be viewed as a mixture of two superfluids, one made up from positive

[2] The problem of the observed highly perfect Planck distribution occurs at a lesser degree in connection with the observed inhomogeneity of the galaxy distribution because the observed root-mean-square fractional temperature fluctuation of the microwave background radiation, less than $10^{-4}$ (soon be improved to less than $10^{-5}$), is difficult to be reconciled with the primordial inhomogeneities which are needed to explain the formation of galaxies.

[3] The idea that the Lorentz group is a dynamic symmetry can be traced back to the pre-Einstein relativity by Fitzgerald, Lorentz and Poincaré, preceded by an even older idea of Voigt [10], with a derivation of the Lorentz invariance as a dynamic symmetry from an underlying Galilei invariance first given by Huntington [11]. More recently the idea has been reconsidered by Janossy [12], Prokhovnik [13], and Wilhelm [14]. In a paper by the author [15], possible consequences of this dynamic interpretation of special relativity for large objects are discussed.

and the other one from negative, densely packed Planck masses. Consistent with the nonrelativistic nature of the model, it has a distinguished reference system in which the two superfluids are at rest. As in Heisenberg's nonlinear spinor theory, the interactions are assumed to be local. The nonlinear operator field equation for the operators $\psi_\pm$, with $\psi_+$ for the positive and $\psi_-$ the negative mass components, is given by [16]

$$i\hbar \frac{\partial \psi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_p} \nabla^2 \psi_\pm + 2\hbar c r_p (\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-) \psi_\pm , \tag{2.1}$$

where $\psi_\pm$, $\psi_\pm^\dagger$ obey the canonical commutation relations

$$[\psi_\pm(\boldsymbol{r})\, \psi_\pm^\dagger(\boldsymbol{r}')] = \delta(\boldsymbol{r} - \boldsymbol{r}') ,$$
$$[\psi_\pm(\boldsymbol{r})\, \psi_\pm(\boldsymbol{r}')] = [\psi_\pm^\dagger(\boldsymbol{r})\, \psi_\pm^\dagger(\boldsymbol{r}')] = 0 . \tag{2.2}$$

In (2.1), $m_p = \sqrt{\hbar c/G}$ is the Planck mass and $r_p = \sqrt{\hbar G/c^3}$ the Planck length, derived from the two Planck relations $G m_p^2 = \hbar c$ and $m_p r_p c = \hbar$, where $G$ is the gravitational constant. As in the theory of superfluidity one can make the Hartree approximation, whereby the field operators are replaced by their expectation values: $\phi_\pm = \langle \psi_\pm \rangle$, $\phi_\pm^* = \langle \psi_\pm^\dagger \rangle$. One then obtains from (2.1) the nonlinear Schrödinger equation

$$i\hbar \frac{\partial \phi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_p} \nabla^2 \phi_\pm + 2\hbar c r_p^2 (\phi_+^* \phi_+ - \phi_-^* \phi_-) \phi_\pm \tag{2.3}$$

describing two superfluids, one possessing positive, and the other one negative mass, coupled with each other through the nonlinear cubic terms. For small amplitudes (2.3) leads to solutions representing longitudinal compressional waves propagating with the velocity of light.

Solutions representing excited states are quantized vortices with a core radius equal to the Planck length. Because an equal number of positive and negative mass vortices does not change the total energy of the two superfluids, there are an infinite number of multiple vortex states of total energy equal to zero. Any one of these states may form a tangle of vortex filaments, sometimes also called a vortex sponge. Because vortices are set into uniform motion under their mutual influence, the vortex filaments collide, snap and are reconnected with other filaments. As computer simulations have shown, such a process leads in its final state to a lattice of vortex rings. For these rings to have a minimum drag on each other, one should expect that an appropriately defined Reynolds number for this vortex lattice must have a critical value. In classical fluid dynamics the drag becomes a minimum for a Reynolds number of the order 250 000, which would imply that the distance of separation between the vortex rings, but also their radius, would have to be about $\sim 10^3 - 10^4$ times larger than the diameter of their cores. With the core radius equal the Planck length, the distance of separation and the ring radius would by order of magnitude be $r_0 \simeq 10^{-30}$–$10^{-29}$ cm.

The Planck masses bound in the vortex filaments execute zero oscillations possessing a kinetic energy of the order $\hbar c/r_p$. These oscillations result in a kinetic energy density within the vortex core, which by order of magnitude is

$$\varepsilon_k \simeq \hbar c / 8\pi r_p^4 . \tag{2.4}$$

The zero point oscillations of the Planck masses confined in the vortex filaments become the source of virtual phonons setting up a force field $\boldsymbol{g}$. To compute its strength we equate the energy density $g^2/8\pi$ of this force field at $r = r_p$ with $\varepsilon_k$ and find

$$\boldsymbol{g} = \sqrt{\hbar c}/r_p^2 = \sqrt{G}\, m_p / r_p^2 . \tag{2.5}$$

Since the phonons propagate with the velocity of light, the zero point oscillations of the Planck masses bound in the vortex filaments become the origin of the gravitational charge $\sqrt{G}\, m_p$, setting up a scalar Newtonian gravitational force field. The phonons, having a cut-off at the wavelength $\lambda \sim r_0$, therefore couple the vortex rings through a Newtonian force field. Through this coupling, the deformation of the vortex lattice can lead to two kinds of fields: One corresponding to a symmetric tensor of deformation which can be identified with Einstein's tensorial gravitational field, and the other one corresponding to an antisymmetric tensor of deformation with Maxwell's vectorial electromagnetic field [17]. Because there is an equal number of positive and negative Planck masses bound in the vortex filaments, it immediately follows that the cosmological constant vanishes exactly.

Under an elliptic deformation the vortex rings have a sharp resonance with an energy equal to

$$E \simeq \pm m_p c^2 (r_p/r_0)^2 \simeq \pm 10^{12}\ \text{GeV} . \tag{2.6}$$

The positive sign applies to the resonances for the vortices of the positive mass superfluid, and the negative sign for the vortices of the negative mass superfluid. A pair composed of a positive and a negative resonance can move through the two component superfluid as an exciton. This motion would be self-accelerating and would follow a straight trajectory would it not be for the static scalar gravitational field set up between these resonances. For masses of opposite sign the energy of this static gravitational field is positive. The composition of the two resonances therefore leads to a configuration consisting of two large masses ($\pm \sim 10^{12}$ GeV) of opposite sign, superimposed by a small positive mass coming from their mutual gravitational interaction. For a mass dipole with a mass monopole superimposed on it, the resulting trajectory is a screwline,with the velocity along the trajectory reaching the velocity of light. It was shown by Hönl and Papapetrou [18] that such a "pole-dipole particle" configuration can reproduce the "Zitterbewegung" derived by Schrödinger [19] from Dirac's equation, and where it is a consequence of an admixture of the negative energy states. Bopp [20] finally was able to derive the Dirac equation from a generalized Hamiltonian mechanic admitting positive as well as negative masses[4].

Assuming that the positive and negative masses are given by the vortex resonance (2.6), an expression for the mass of the Dirac particle can then be computed, which turns out to be

$$m = 2\,(r_\mathrm{p}/r_0)^6\, m_\mathrm{p}\,. \tag{2.7}$$

For the value $r_0/r_\mathrm{p} \simeq 5000$, conjectured from the existence of a minimum drag Reynolds number for the vortex lattice, it reproduces remarkably well the mass of a typical fermion.

## 3. Fermion Production During the Big Bang

If in the course of the big bang fermions are formed, the total energy of the Planck aether must remain zero because in its groundstate the Planck aether has zero energy. Prior to the formation of the fermions one had $\Omega = 1$, and with the total energy remaining equal to zero, the value $\Omega = 1$ is therefore conserved during the formation of the fermions. The formation of the fermions must be accompanied by the release of a considerable amount of energy in the form of heat. Part of this energy can go into electromagnetic and gravitational waves, but also into neutrinos. As the energy density of the cosmic microwave background radiation shows, the contribution of the electromagnetic energy to the value of $\Omega$ is negligible, quite apart from the fact that it is not the kind of cold dark matter the missing mass should consist of. Because the Planck aether is a superfluid having an energy gap, energy dissipated during the formation of the fermions can also go into an excited state above this gap. This behavior is most easily understood from the energy spectrum of a superfluid shown in Figure 1. For sufficiently low energies it is described by a spectrum consisting of phonons, as in a solid. But for energies near the upper cut-off, which in our model is the Planck energy $m_\mathrm{p} c^2$, the spectrum has a dip. Quasiparticles described by this part of the energy spectrum are called rotons, and since at the minimum of the dip $\mathrm{d}E/\mathrm{d}k = 0$, the rotons must have a small velocity. The height of the minimum of the dip can be viewed as an energy gap $\Delta$, which by order of magnitude is $\Delta \sim m_\mathrm{p} c^2$. With the width $\Delta k$ of the dip of the order $\sim r_\mathrm{p}^{-1}$, the excited state of rotons behaves like a fluid of free particles, with each roton mas approximately equal to $m_\mathrm{p}$ and having the same gravitational charge $\sqrt{G}\, m_\mathrm{p}$ as Planck masses confined in a vortex filament.

The property of a fluid composed of rotons follows from the dispersion relation of the superfluid Planck aether. Near the minimum at $k = k_0$ it has the form

$$\hbar\omega = \Delta + \frac{\hbar^2 (k - k_0)^2}{2 m_\mathrm{r}}\,, \tag{3.1}$$

where $m_\mathrm{r} \lesssim m_\mathrm{p}$ is the roton mass. The rotons to the right of the minimum at $k = k_0$ are called $\mathrm{R}^+$ rotons, those to the left $\mathrm{R}^-$ rotons. The velocity of the rotons is the group velocity

$$v = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{\hbar (k - k_0)}{m_\mathrm{r}}\,, \tag{3.2}$$

and their momentum

$$p = p_0 + m v\,, \tag{3.3}$$

where $p_0 = \hbar k_0$, is the momentum at $k = k_0$ where $v = 0$.

It follows that a fluid composed of rotons has a finite pressure even if the velocity of the rotons vanishes. Because a finite pressure can mimic a cosmological constant, a roton fluid can mimic a cold gas, composed of low velocity particles with a superimposed cosmological constant.

[4] With Bopp's generalized nonlinear mechanic one can even predict a maximum of 4 particle families [21], in good agreement with the width of the $Z$ particle and the measured helium abundancies, leading to three, but not more than four families.

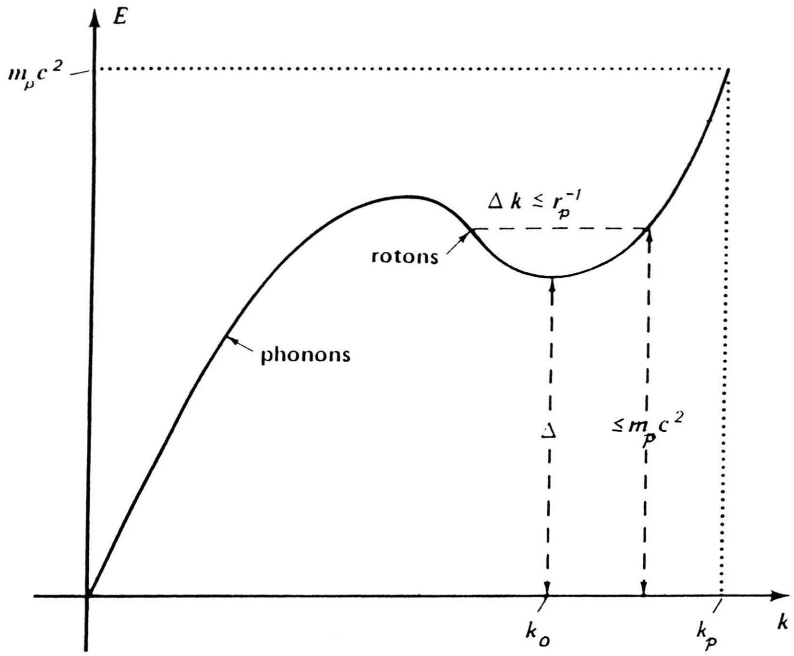


Fig. 1. The phonon-roton energy spectrum of the hypothetical Planck aether.

A very good match for the observed matter distribution in the universe is obtained if the contribution of a cosmological constant ($\Lambda$) to $\Omega$ would be $\Omega_\Lambda \simeq 0.8 \pm 0.1$, and of cold dark matter (CDM) to $\Omega_{CDM} \simeq 0.2 \pm 0.1$, making $\Omega_\Lambda + \Omega_{CDM} = 1$ [6]. This twofold requirement, needed to reach $\Omega = 1$, is the so-called $\Omega$-problem [1]. It is the strange property of the rotons, that they are able to account for both contributions, making them a unique candidate to account for the missing mass.

According to (3.1), the total energy of a roton is

$$E = \Delta + \tfrac{1}{2} m_r v^2 . \tag{3.4}$$

For relativistic roton energies, it can be written as

$$E = (\Delta - m_r c^2) + \frac{m_r c^2}{\sqrt{1 - v^2/c^2}} \tag{3.5}$$

with the first term making the contribution to $\Omega_\Lambda$ and the second one to $\Omega_{CDM}$. To obtain values for $\Delta$ and $m_r$, needed to estimate the contributions $\Omega_\Lambda$ and $\Omega_{CDM}$ to $\Omega$, we assume that the phonon-roton spectrum is universal. Under this assumption we can use the phonon-roton spectrum obtained from measurements in superfluid liquid helium by Henshaw and Woods [22]. According to these measurements one has $\Delta \simeq 0.52\, m_p c^2$ (equating the Debye-energy with the Planck energy $m_p c^2$) and $m_r \simeq 0.16\, m_p$ (equating the helium mass with the Planck mass $m_p$). Inserting these values into (3.5), the energy of a Planck aether roton therefore is

$$\frac{E}{m_p c^2} \simeq 0.36 + \frac{0.16}{\sqrt{1 - v^2/c^2}} . \tag{3.6}$$

From this expression one immediately sees that $\Omega_\Lambda \simeq 0.7$ and $\Omega_{CDM} \simeq 0.3$, with $\Omega_\Lambda + \Omega_{CDM} = 1$. This is a surprisingly good agreement with the empirical values $\Omega_\Lambda \simeq 0.8 \pm 0.1$ and $\Omega_{CDM} \simeq 0.2 \pm 0.1$, to explain the matter distribution in the universe [5].

A roton number density equal to $n_r \cong 2 \times 10^{-25}$ cm$^{-3}$, with a roton mass $m_r \lesssim m_p \simeq 2.2 \times 10^{-5}$ g, would be sufficient to reach the critical density $\varrho \simeq 4.5 \times 10^{-30}$ g/cm$^3$, needed to make $\Omega = 1$. At this number density the distance between two (Planck mass) rotons would be $n_r^{-1/3} \simeq 2800$ km. This low number density, combined with their weak gravitational interaction, would make a direct detection of the Planck aether rotons very difficult.

[5] The values for $\Omega_\Lambda$ and $\Omega_{CDM}$ are actually somewhat smaller to make room for cold baryonic matter, having a fraction less than 0.1. The contribution coming from hot baryonic matter is much smaller still, possibly as small as $\sim 10^{-3}$.

## 4. The Normal Component of the Planck Aether

As is known from the theory of superfluidity [23], the existence of a normal component leads to a longitudinal wave mode called second sound. Compared with the propagation velocity of the first sound, the second sound has a propagation velocity smaller by the factor of $1/\sqrt{3}$. In the Planck aether the first sound propagates with the velocity of light. The propagation velocity of the second sound therefore is

$$c_s = c/\sqrt{3} = 173\,000 \text{ km/sec}. \tag{4.1}$$

The second sound consists of propagating temperature oscillations, which behave like scalar photons in a medium with a refractive index equal to $\sqrt{3}$. During the cosmic expansion, the second sound must therefore have cooled down by the same proportion as the temperature of the cosmic microwave background radiation. Because the second sound consists of temperature or entropy waves, it should smoothed out any temperature inhomogeneities during the initial phase of the cosmic expansion. It therefore can account for very smooth initial conditions, mimicking the inflationary scenario of the standard model.

In the Planck aether model, the electromagnetic interaction sets in below the GUT energy at $\sim 10^{16}$ GeV. In the region between the Planck and GUT energy, there are only scalar gravitons and second sound phonons. The smoothing out can in this energy interval take place through second sound phonons, rather than through inflation.

## 5. The Rotation Curves of Disc Galaxis

Because the roton masses of the normal component are large, their velocity relative to the galaxies must be small. Rotons in the intergalactic space are falling towards the center of galaxies, and because of their very small cross section, they are thereafter flowing radially out. If their density at a large distance $r_0$, measured from the center of attraction, is $\varrho_0$ and their velocity at this distance $v_0$, their density $\varrho$ at the distance $r$ is

$$\varrho = \varrho_0 \left(\frac{r_0}{r}\right)^2 \left(\frac{v_0}{v}\right). \tag{5.1}$$

We assume that the value of $\varrho_0$ is equal the critical density to obtain $\Omega = 1$.

With (5.1), the gravitational potential is obtained from Poisson's equation in spherical coordinates,

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[r^2 \frac{\mathrm{d}\phi}{\mathrm{d}r}\right] = 4\pi G \varrho. \tag{5.2}$$

To compute the velocity for the rotons falling into the gravitational potential, we have to consider the dynamic behavior obtained from their dispersion relation (3.1), with the roton velocity given by their group velocity $\mathrm{d}\omega/\mathrm{d}k$.

The equation of motion of the rotons falling into a centrally symmetric gravitational potential $\phi$ therefore is

$$m_r \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}\omega}{\mathrm{d}k}\right) = -m_r \frac{\mathrm{d}\phi}{\mathrm{d}r}. \tag{5.3}$$

With $\mathrm{d}/\mathrm{d}t = (\mathrm{d}r/\mathrm{d}t)\,\mathrm{d}/\mathrm{d}r = v\,\mathrm{d}/\mathrm{d}r = (\mathrm{d}\omega/\mathrm{d}k)\,\mathrm{d}/\mathrm{d}r$ one obtains that from (5.3) by integration

$$(\mathrm{d}\omega/\mathrm{d}k)^2 = v^2 = -2\phi, \tag{5.4}$$

both valid for the $R^+$ and $R^-$ rotons, with $\phi$ the gravitational potential at the distance $r$. In spite of their unusual dispersion relation, the rotons, like any other form of matter, obey the equivalence principle.

At the distance $r = r_0$, where $\phi = \phi_0$, the velocity $v = v_0$ shall be related to Hubble's law

$$v_0 = H r_0, \tag{5.5}$$

where

$$H = \sqrt{8\pi G \varrho_0/3} \tag{5.6}$$

is the Hubble constant with $\varrho_0$ set equal to the value for which $\Omega = 1$. From (5.4)–(5.6), one has

$$\phi_0 = -(4\pi/3)\, G \varrho_0 r_0^2. \tag{5.7}$$

With (5.1) and (5.7), (5.2) becomes

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[r^2 \frac{\mathrm{d}\phi}{\mathrm{d}r}\right] = -3\phi_0 \sqrt{\frac{\phi_0}{\phi}}. \tag{5.8}$$

Putting $r/r_0 = x$, $\phi/\phi_0 = y$, (5.8) can be written as

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[x^2 \frac{\mathrm{d}y}{\mathrm{d}x}\right] = -\frac{3}{\sqrt{y}} \tag{5.9}$$

with the boundary condition $y = 1$ at $x = 1$. Introducing the new variable

$$u = 1 - 3\ln x, \tag{5.10}$$

(5.9) first becomes

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[x\frac{\mathrm{d}y}{\mathrm{d}u}\right]=\frac{1}{\sqrt{y}} \tag{5.11}$$

with $y=1$ at $u=1$. With the substitution (5.10), (5.11) finally becomes

$$-3\frac{\mathrm{d}^2y}{\mathrm{d}u^2}+\frac{\mathrm{d}y}{\mathrm{d}u}=\frac{1}{\sqrt{y}}, \tag{5.12}$$

For (5.12), we seek solutions of the form

$$y=\sum_n a_n u^n \tag{5.13}$$

For $x\to 0$, that is for $r\to 0$, one has $u\to\infty$. Therefore, if $u\to\infty$, only the highest power $n$ contributes to the solution of (5.12).

In the limit $u\to\infty$, $\mathrm{d}^2y/\mathrm{d}u^2$ can therefore be neglected against $\mathrm{d}y/\mathrm{d}u$, and one obtains

$$\lim_{u\to\infty} y=(3/2)^{2/3}\,u^{2/3}\,. \tag{5.14}$$

Hence

$$\lim_{r\to 0}\phi/\phi_0=[\tfrac{3}{2}(1-3\ln(r/r_0))]^{2/3}\,. \tag{5.15}$$

By inserting into (5.12) for $\mathrm{d}^2y/\mathrm{d}u^2$ and $\sqrt{y}$ the asymptotic solution (5.14), one obtains the approximate differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}u}\simeq\left(\frac{2}{3}\right)^{1/3}[u^{-1/3}-u^{-4/3}]\,. \tag{5.16}$$

For $u\to\infty$ it has the same asymptotic solution as the exact differential equation. It therefore describes the departure from this asymptotic solution. By integration of (5.16), one obtains the approximate solution

$$y\simeq\left(\frac{3}{2}\right)^{2/3}u^{2/3}\left[1+\frac{2}{u}\right]. \tag{5.17}$$

Higher approximations obtained by this iteration procedure lead to solutions of the form

$$y\simeq\left(\frac{3}{2}\right)^{2/3}u^{2/3}\sum_{n=0}^{\infty}a_n u^{-n}\,. \tag{5.18}$$

In applying these results to the rotation curves of disc galaxies where $r\ll r_0$, it suffices to take the first term in the expansion of (5.18), which is the asymptotic solution (5.14):

$$\frac{\mathrm{d}\phi}{\mathrm{d}r}=-\frac{2\,(3/2)^{2/3}}{[1-3\ln(r/r_0)]^{1/3}}\,\frac{\phi_0}{r}\,. \tag{5.19}$$

For the rotation curves, (5.19) has to be equated with the centrifugal force per unit mass, $V^2/r$, where $V$ is the azimuthal velocity. One has

$$V=\frac{\text{const}}{[1-3\ln(r/r_0)]^{1/6}}\,. \tag{5.20}$$

For $r\ll r_0$, this is

$$V\simeq\text{const}\,[\ln(r_0/r)^3]^{-1/6}\,, \tag{5.21}$$

showing a weak logarithmic dependence of $V$ on $r$, in qualitative agreement with the observed flat rotation curves.

With the help of (5.6) and (5.7) one obtains

$$V=\frac{(3/2)^{1/3}\,H\,r_0}{[1-\ln(r/r_0)^3]^{1/6}}\,. \tag{5.22}$$

With $H\simeq 150$ km/sec/$10^6$ light years, $r_0\simeq 10^6$ light years, $r_1\simeq 10^5$ light years (typical radius of a disc galaxy), one finds $V\simeq 120$ km/sec, in good agreement with the observed rotation velocities.

## 6. Gravitational Collapse

The Planck aether model leads in the asymptotic limit for low energies to Einstein's field equations [17]. At very high energies the gravitational waves derived from this model have a cut-off length of the order $r_0$. Since this length corresponds to an energy of $\sim 10^{16}$ GeV, Einstein's equations should break down approaching this energy. The Einstein gravitational field Lagrangian

$$L_g=\sqrt{-g}\,R \tag{6.1}$$

should therefore be replaced by a more general gravitational Lagrangian density incorporating the cut-off length $r_0$.

A hint for obtaining such a Lagrange density is provided by compressible fluid dynamics, where for high densities the ideal gas equation has to be replaced by the Van der Waals equation of state. With the velocity potential $v=-\text{grad}\,\phi$, the fluid dynamic equations for an ideal gas can be derived from the Lagrange density

$$L_f=\varrho\,\phi-\frac{\varrho}{2}(\text{grad}\,\phi)^2-\varrho\,i\,, \tag{6.2}$$

where $i$ is the enthalpy. Variation with regared to $\varrho$ leads to Bernoulli's equation

$$\frac{\partial \phi}{\partial t} = \tfrac{1}{2} v^2 + i\,, \tag{6.3}$$

and the variation with regard to $\phi$ leads to the continuity equation

$$\frac{\partial \varrho}{\partial t} + \operatorname{div}(\varrho \boldsymbol{v}) = 0\,. \tag{6.4}$$

In case of an ideal gas, one has

$$\varrho i = \varrho c_p T = [\gamma/(\gamma-1)]\, p \tag{6.5}$$

($\gamma = c_p/c_v$; $c_p$, $c_v$ specific heat at constant pressure and density, $p$ pressure, $T$ absolute temperature). For a Van der Waals equation of state (without an attractive intermolecular potential), one has to make in the ideal gas equation the substitution

$$\frac{p}{\varrho} \rightarrow \frac{p}{\varrho(1-\varrho/\varrho_0)} \tag{6.6}$$

A fluid dynamic Lagrangian, with the replacement (6.6), would lead for $\varrho \rightarrow \varrho_0$ to $p \rightarrow \infty$. The Einstein-Hilbert gravitational field Lagrangian in the presence of matter leads for $T \rightarrow \infty$ to $R \rightarrow \infty$, very much as in an ideal gas where for $\varrho \rightarrow \infty$, $p \rightarrow \infty$. It is therefore plausible to propose a generalized gravitational field Lagrangian, employing the same line of thought as for the Van der Waals equation, by replacing (6.1) with

$$L_g = \sqrt{-g}\, R/(1 - r_0^2 R)\,. \tag{6.7}$$

This Lagrangian has the desired property that for $R \rightarrow 1/r_0^2$, one has $L_g \rightarrow \infty$. Because of this property, we expect that the Lagrange density (6.7) does not lead to a singularity in the course of gravitational collapse, as does the Einstein Lagrangian, which for $L_g \rightarrow \infty$ leads to $R \rightarrow \infty$.

Nonlinear gravitational field Lagrangians of the general form

$$L_g = \sqrt{-g}\, f(R) \tag{6.8}$$

have been studied by Buchdahl [24]. Together with the variation of the matter Lagrangian, they lead to the following field equations [6] ($\varkappa = 8\pi G/c^4$):

$$f'(R)\, R_{ik} - \tfrac{1}{2}\, g_{ik}\, f(R) = \varkappa T_{ik}\,,$$
$$(f'(R)\, g^{kl} \sqrt{-g})_{;m} = 0\,. \tag{6.9}$$

For $f(R) = R$, these are Einstein's field equations. Contracting the first equation of (6.9) one has

$$R f'(R) - 2 f(R) = \varkappa T\,. \tag{6.10}$$

For $f(R) = R/(1 - r_0^2 R)$ one finds

$$\frac{R}{(1 - r_0^2 R)^2} - \frac{2R}{1 - r_0^2 R} = \varkappa T\,. \tag{6.11}$$

In the limit $r_0 = 0$, one obtains the contracted Einstein equation

$$R = -\varkappa T\,. \tag{6.12}$$

Applied to a liquid for which

$$T = -\varrho c^2 + 3p \tag{6.13}$$

one has

$$R = \varkappa(\varrho c^2 - 3p)\,, \tag{6.14}$$

showing that during gravitational collapse, where $\varrho c^2 \rightarrow \infty$, one has with necessity $R \rightarrow \infty$. It is here where the problem of the space-time singularity has its root.

A different situation exists if (6.12) is replaced by (6.11). There $T \rightarrow \infty$ for $R \rightarrow 1/r_0^2$. Putting $r_0^2 R = x$ and $\varkappa r_0^2 T = y$, one has for (6.11)

$$\frac{x}{(1-x)^2} - \frac{2x}{1-x} = y\,. \tag{6.15}$$

The inverse function describes the dependence $R(T)$. The function $y(x)$ is plotted in Figure 2. Its properties are that $y = 0$ for $x = 0$ and $x = 1/2$; $dy/dx = -1$ for $x = 0$; and $dy/dx = 0$ for $x = 1/3$, $y = -1/4$. In the limit $r_0 \rightarrow 0$, valid for Einstein's field equation, (6.10) becomes $y = -x$. Whereas in Einstein's limiting case $R \rightarrow \infty$ for $T \rightarrow -\infty$, $R$ and $T$ are now limited by $R \leq 1/r_0^2$, and $T \geq -1/4\varkappa r_0^2$. Because $T = -\varrho c^2 + 3p$, it follows that with increasing energy density, $R$ increases monotonically up to the value $R = 1/3 r_0^2$, at which $T = T_{\min} = -1/4 \varkappa r_0^2$. After reaching this point, $R$ still can increase, but only if $T$ from there on increases until $R = 1/2 r_0^2$, and where $T = 0$. Because for electromagnetic radiation one has $T = 0$, the only way our Van der Waals type Lagrangian can follow the collapse is by conversion of matter into

[6] Dr. H. J. Schmidt [25] has pointed out to me that in general these field equations are only true if $T$ and $R$ are constant. Since our Lagrangian (6.7) should be understood as a model, conveniently containing a cut-off at $R \sim 1/r_0^2$, a discussion based on this assumption should give us a qualitative understanding of how the singularity can be avoided.

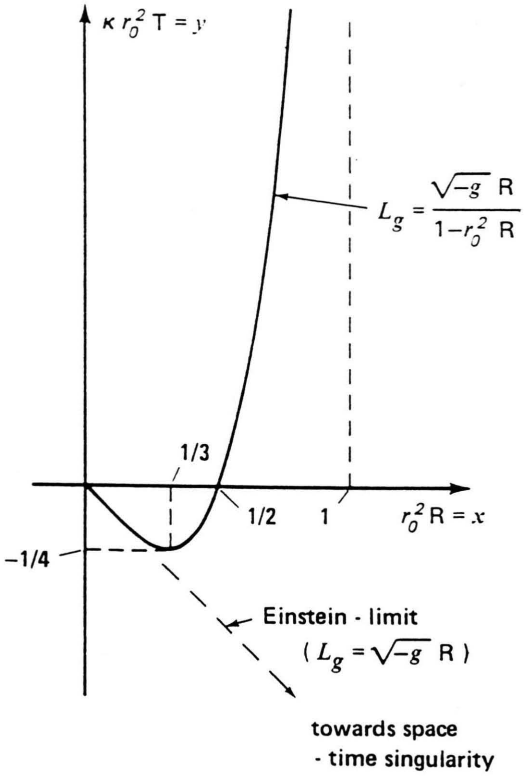


Fig. 2. The dependence of $T$ on $R$ with an Einstein-Hilbert gravitational field Lagrangian and a Van der Waals type gravitational field Lagrangian.

radiation. It would take place above a certain critical value for the trace of the energy momentum tensor given by

$$(\varrho c^2 - 3p)_{\rm crit} = 1/4\varkappa r_0^2 . \tag{6.16}$$

When $R$ has reached the maximum value $R_{\max} = 1/2 r_0^2$, where $T = 0$, all the matter has been converted into radiation.

Because the term $r_0^2 R$ is normally very small, except near the singularity, we may study the rise in the energy density leading to this conversion into radiation by considering Schwarzschild's interior solution. The line element of this solution, valid for an incompressible liquid of density $\varrho$, is given by [26]

$$ds^2 = -\frac{dr^2}{1 - r^2/R_0^2} - r^2 (d\theta^2 + \sin^2\theta\, d\phi^2) + \left[\frac{3}{2}\sqrt{1 - \frac{r_1^2}{R_0^2}} - \frac{1}{2}\sqrt{1 - \frac{r^2}{R_0^2}}\right]^2 c^2\, dt^2 , \tag{6.17}$$

where

$$R_0^2 = \frac{3c^2}{8\pi G \varrho} , \quad m = (4\pi/3)\, \varrho\, r_1^3 . \tag{6.18}$$

$m$ is the total mass of the liquid sphere of radius $r = r_1$.

The pressure in the sphere is given by

$$p = \varrho c^2 \frac{\sqrt{1 - r^2/R_0^2} - \sqrt{1 - r_1^2/R_0^2}}{3\sqrt{1 - r_1^2/R_0^2} - \sqrt{1 - r^2/R_0^2}} . \tag{6.19}$$

For a vanishing denominator in (6.19) the pressure diverges. To keep the pressure finite, the inequality

$$3\sqrt{1 - r_1^2/R_0^2} > \sqrt{1 - r^2/R_0^2} \tag{6.20}$$

must be satisfied. Putting $x = r_1/R_0$, $y = r/R_0$, the inequality can be written as

$$y > \sqrt{9x^2 - 8} , \quad x > \sqrt{8/9} , \qquad y \geq 0 , \quad x < \sqrt{8/9} . \tag{6.21}$$

The singularity develops for $x \geq \sqrt{8/9}$, starting from $y = 0$, that is from $r = 0$ radially outwards with increasing $x$, by adding mass at constant density. At $y = 0$ (that is at $r = 0$) the pressure rises as

$$p_0 = \varrho c^2 \frac{1 - \sqrt{1 - x^2}}{3\sqrt{1 - x^2} - 1} . \tag{6.22}$$

Putting $x = x_0 - \varepsilon$, $x_0 = \sqrt{8/9}$, one obtains in approaching $x \to x_0$

$$p_0 \to (1/9\sqrt{2})\, \varrho c^2/\varepsilon . \tag{6.23}$$

The Schwarzschild interior solution should become invalid if by order of magnitude the pressure in the relativistic limit becomes

$$p = u/3 \gtrsim 1/3\varkappa r_0^2 . \tag{6.24}$$

Defining a critical mass density

$$\varrho_c = u/c^2 = c^2/(8\pi G r_0^2) \tag{6.25}$$

above which Einstein's equations become invalid, and introducing the Planck density $\varrho_p = m_p/r_p^3$, (6.25) can be written as

$$\varrho_c/\varrho_p = (1/8\pi)\,(r_p/r_0)^2 . \tag{6.26}$$

Since $r_p \sim 10^{-33}$ cm, $\varrho_p \sim 10^{94}$ g/cm$^3$, $r_0/r_p \sim 5 \times 10^3$, one has $\varrho_c \sim 10^{85}$ g/cm$^3$. This enormous density is brought into better perspective if one realizes that it is by order of magnitude the same density reached in a sphere of mass $m_0 \sim \varrho_c r_0^3$ having the Schwarzschild radius $r_0$.

Another instructive way of looking at (6.26) is, that $\varrho_c$ is by order of magnitude equal to the mass density of compactified vortex filaments, which is

$$\varrho_v \sim m_p(r_0/r_p)/r_0^3 = \varrho_p(r_p/r_0)^2 . \quad (6.27)$$

Since in the Planck aether model Dirac spinors are excitons made up from the positive and negative resonance energy of the ring vortices, it is plausible that if these excitons are compressed to such a state of compactification, whereby they come into contact with each other, their mutual disturbing influence should lead to their disintegration. [7]

Very much as in classical gas dynamics, where a finite mean free path prevents the development of a singularity in the center of a convergent shock wave, the finite vortex ring radius $r_0$ limits here the growth towards a space-time singularity.

The conversion of mass into energy in the center of a collapsing star would have, of course, far reaching consequences and could explain the large energy release in quasars. The described prevention for the occurrence of the space-time singularity would outlaw the establishment of mini-black-holes having masses larger than

$$m \simeq r_0 c^2/G \sim 1 \text{ g} . \quad (6.28)$$

It therefore appears that mini-black-holes left over from the primordial fire ball are unlikely candidates for the missing dark matter, because mini-black-holes with a mass of $\lesssim 1$ g would have disintegrated a long time ago by Hawking radiation.

Finally, we would like to remark that the formation of a naked singularity in Einsteinian general relativity seems to have been recently confirmed in numerical calculations on a supercomputer by Shapiro and Teukolsky [28]. If supported by further more detailed calculations, this result would show that Einstein's equations must break down in approaching the singularity. Our model, however, shows that Einstein's equations would still hold up very close to the singularity, and would only have to be replaced by field equations derived from a more general Lagrangian, taking into account a smallest wavelength at $\lesssim 10^{16}$ GeV, thereby avoiding the occurrence of a singularity.

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[7] It has been shown by Dehnen et al. [27] that SU5-GUT models would lead to a conversion of baryons into leptons, and hence, in the course of gravitational collapse of mass into radiation. The pressure at which this conversion would occur is the same as in our model, corresponding to the GUT energy of $\sim 10^{16}$ GeV.

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